

Fun Facts First:

the HP32E uses two pettily different versions of the constant $\frac{1}{\sqrt{2\pi}}$,

0,398942280444 and
0,398942280385.

Q^{-1} Function of HP32E

The HP32E is one of the few models that offers besides the Q function, the cumulative distribution function (CDF) of the standard normal distribution, also its reversal, Q^{-1} , the inverse cumulative distribution function, aka *probit*, or the quantile function associated with the standard normal distribution. (For its graph [click here](#).)

In other words, for input x ...

$$p = \Phi(z) \quad \text{on 32E: } Q(x)$$

$$z_p = \Phi^{-1}(p) \quad \text{on 32E: } Q^{-1}(x) \quad \text{or: } x \stackrel{!}{=} Q(z)$$

The Details

Analysing trace logs made with Tony's [HP Classic Calculator Emulator Plus](#) I found, the HP32E uses its approximations of Q to solve numerically $x - Q(z) = 0$ for z , thus $Q^{-1}(x)$. The Q^{-1} function is split in three parts (special cases input <0 , $=0$, $=1$, and >1 are sorted out first), one for the input range $]0..0,1[$ (part 1) and next from $[0,1..0,9[$ (part 2). The third range $[0,9..1[$ is "folded down" to part 1 using the function's symmetry. (The limits 0,1 and 0,9 correspond to those $\pm 1,28$ of Q , dividing the scope correspondingly.)

Q^{-1} Part 1 and 3

Prepare first loop...

if $0 < x < 0,1 \Rightarrow$ part 1,

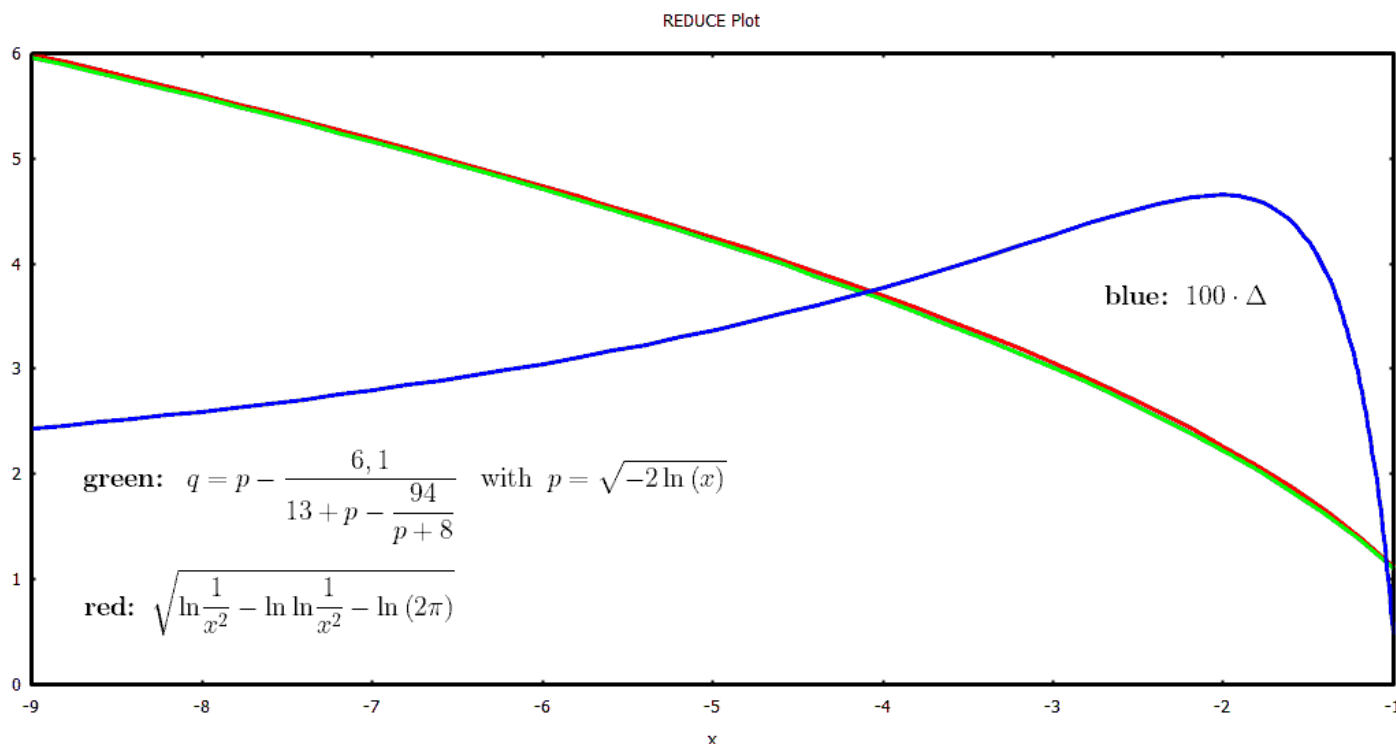
if $0,9 \leq x < 1 \Rightarrow$ part 3, remember for later

if part 3 set $x := 1 - x$

$$p = \sqrt{-\ln(x^2)}$$

$$q = p - \frac{6,1}{13 + p - \frac{94}{p+8}} \approx \sqrt{\ln \frac{1}{x^2} - \ln \ln \frac{1}{x^2} - \ln(2\pi)} + o(1)$$

— it's *amazing* close (or just sufficing?) to the [asymptotic expansion](#) (for small input).



But then, alas:

$z_0 = q^2/2$ the initial guess, **an error**, should be q
 $h_0 = e^{z_0}$ when righting above error use $e^{q^2/2}$
 begin...
 r_n as in "Q.part 2" but computed with z_{n-1} instead of $|x|$
 $s_n = 1/(z_{n-1} + b_2 + r_n)$ with $b_2 = -3,8052E-8$
 $t_n = x \cdot h_{n-1}/a_1$ with $a_1 = 0,398942280444$
 $k_n = z_{n-1}(t_n - s_n)$ or: $\sqrt{2\pi} \cdot h \cdot z \cdot (x - Q(z))$
 $h_n = h_{n-1}/(1 + k_n)$
 $z_n = \sqrt{2 \ln(h_n)}$ or: $\sqrt{z^2 - 2 \ln(1 + k)}$
 loop while $|k_n| > 1E-10$
 if part 3 set $z := -z$

Part 2

For input $0, 1 \leq x < 0, 9$ it goes like this:

Prepare first loop...
 $s = x - 0, 5$
 $z_0 = s/b_1$ the initial guess, $b_1 = 0,398942280385$
 begin...
 $t_n = z_{n-1}^2/2$
 $u_n = Q(z_{n-1}) - 0, 5$
 $w_n = (u_n - s) \cdot e^{t_n}/b_1$
 $z_n = z_{n-1} - w_n$
 loop while $|w_n| > 1E-10$

Lessons Learned?

Trace logs enable to detect the inbuilt algorithms rather mechanically, but to recognize the maths principle behind a procedure needs a bit more than just know-how, some know-why would help.

Part 2 is Newton's method, completely as in every maths standard textbook. e^{t_n}/b_1 is the reciprocal value of the Q function's derivative. Got it, ticked it.

For part 1 – sorry, no idea (yet). I am able to spot the $(x - Q(z))$ step, I am able to copy the procedure to another system (and it works), I did find an error in first valuation of z (what is ironed out with one or two more iterations), but in contrast to part 2 I do miss the eureka moment.